

Equation (21) gives the displacement over the surface of the crack in the linear viscoelastic medium for a general loading  $S(\rho, t)$ . The corresponding stress component  $\sigma_{\theta z}(\rho, 0, t)$  can be evaluated by employing Eq. (21) in Eq. (11). If, in particular,  $S(\rho, t)$  is independent of  $\rho$ , i.e.,  $S(\rho, t) = S(t)$  then from Eq. (21) we find

$$u_0(\rho, 0, t) = S(t) \left[ \rho \log \left\{ \frac{a(t) + [a^2(t) - \rho^2]^{1/2}}{\rho} \right\} G_1^{-1}(0) \right] + \int_0^t S(\tau) \rho \log \left\{ \frac{a(t-\tau) + [a^2(t-\tau) - \rho^2]^{1/2}}{\rho} \right\} \frac{\partial G_1^{-1}(\tau)}{\partial \tau} d\tau \quad \rho \leq a(t) \quad (22)$$

On substituting Eq. (21) in Eq. (11) and interchanging the order of integration we obtain

$$\sigma_{\theta z}(\rho, 0, t) = -(1/2) \int_0^t G_1(t-\tau) \frac{\partial}{\partial \tau} \int_0^\tau G_1^{-1}(\tau-\rho) S(\tau) \frac{\partial}{\partial \rho} \times \int_0^\infty \xi^2 J_1(\xi \rho) d\xi \int_0^{a(\rho)} d\eta \int_0^\eta \frac{\lambda^2 J_1(\xi \lambda)}{(\eta^2 - \lambda^2)^{1/2}} d\lambda d\rho d\tau \quad (23)$$

On using Eqs. (14) and (19) in the above equation and further simplifying, we finally obtain

$$\sigma_{\theta z}(\rho, 0, t) = -S(t) \left[ \int_0^{a(t)} \frac{\eta H(\rho - \eta)}{\rho(\rho^2 - \eta^2)^{1/2}} d\eta - (1/2) \frac{a^2(t) H[\rho - a(t)]}{\rho[\rho^2 - a^2(t)]^{1/2}} \right] \quad (24)$$

For  $\rho \leq a(t)$ , Eq. (24) reduces to  $\sigma_{\theta z}(\rho, 0, t) = -S(t)$  as should be expected. For  $\rho > a(t)$ , it can be further simplified to give

$$\sigma_{\theta z}(\rho, 0, t) = \frac{S(t)}{\rho} [\rho^2 - a^2(t)]^{1/2} - S(t) + \frac{S(t)a^2(t)}{2\rho[\rho^2 - a^2(t)]^{1/2}} \quad \rho > a(t) \quad (25)$$

On returning to the original problem for which the boundary condition in Eq. (4) is to be satisfied, we find

$$\sigma_{\theta z}(\rho, 0, t) = \frac{S(t)}{\rho} [\rho^2 - a^2(t)]^{1/2} + \frac{S(t)}{2\rho} \frac{a^2(t)}{[\rho^2 - a^2(t)]^{1/2}} \quad \rho > a(t) \\ = 0 \quad \rho \leq a(t) \quad (26)$$

Equation (22) gives the displacement over the surface of the crack for the loading,  $S(t)$ . It reduces to the expression of the classical elasticity theory,<sup>6</sup> when the usual limits of passing from viscoelasticity to classical elasticity are employed. Equation (26) is the expression for the stress component  $\sigma_{\theta z}(\rho, 0, t)$  in the plane of the crack, and a comparison of it with the one obtained in classical elasticity theory<sup>6</sup> indicates that there is no effect of viscoelasticity in this quantity.

The stress intensity factor  $N(t)$ , defined by the equation

$$N(t) = \lim_{\rho \rightarrow a^+(t)} \{2[\rho - a(t)]\}^{1/2} \sigma_{\theta z}(\rho, 0, t) \quad (27)$$

turns out to be

$$N(t) = S(t)/2 \{a(t)\}^{1/2} \quad (28)$$

Also, if we define the work done in opening the crack by

$$W(t) = 2\pi \int_0^{a(t)} \rho S(t) u_0(\rho, 0, t) d\rho \quad (29)$$

by the loading  $S(t)$ , it follows from Eq. (22) and Eq. (29) that

$$W(t) = \frac{\pi^2 S^2(t) a^3(t)}{6G_1(0)} + \frac{\pi^2}{6} \int_0^t S^2(\tau) a^3(t-\tau) \frac{\partial}{\partial \tau} G_1^{-1}(\tau) d\tau \quad (30)$$

In the usual limits, if we identify  $G_1(0)$  by  $2\mu$ , we note that first expression on the right-hand side of Eq. (30) is the same as in the classical elasticity theory.<sup>6</sup> The second expression on the right-hand side of Eq. (30) therefore represents the excess amount of work required due to the energy dissipation in the viscoelastic medium.

In order to have some quantitative feeling about the above results we now assume that the viscoelastic medium is a Maxwell material whose behavior is characterized by

$$G_1(t) = G_0 e^{-t/\tau_0} \quad G_1^{-1}(t) = \left( \frac{1}{G_0} \right) \left( 1 + \frac{t}{\tau_0} \right) \quad (31)$$

Hence, it follows that

$$G_1^{-1}(0) = \frac{1}{G_0} \quad \frac{\partial}{\partial \tau} G_1^{-1}(\tau) = (1/G_0 \tau_0) \quad (32)$$

On employing Eq. (32) in Eq. (22), we obtain

$$u_0(\rho, 0, t) = \frac{S(t)}{G_0} \rho \log \left[ \frac{a(t) + \{a^2(t) - \rho^2\}^{1/2}}{\rho} \right] + \frac{1}{G_0 \tau_0} \int_0^t S(\tau) \rho \log \left[ \frac{a(\theta) + \{a^2(\theta) - \rho^2\}^{1/2}}{\rho} \right] d\theta \quad \rho \leq a(t) \quad (33)$$

where a change of variable has been made in writing Eq. (33). Similarly, Eq. (30) reduces to

$$W(t) = \frac{\pi^2 S^2(t) a^3(t)}{6G_0} + \frac{\pi^2}{6G_0 \tau_0} \int_0^t S^2(\theta) a^3(\theta) d\theta \quad (34)$$

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## Surface Heat Transfer in the Flow of Dissociated Nitrogen

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THE possibility of determining the freestream atom concentrations by heat-transfer measurements at surfaces of known catalyticities was investigated by a number of authors. A review of these studies was given by Chung.<sup>1</sup> The known calculations determining the effect of variable wall catalyticities on heat transfer are based on the frozen flow assumption which simplifies the mathematics of the problem appreciably. A double surface gage with a noncatalytic and a highly catalytic surface located one after each other on a flat plate was numerically investigated by Hayday and McGraw,<sup>2</sup> where variable transport properties were taken into account.

The accuracy of such a diagnosis depends much on the possibility of realizing a flow configuration, where the gas phase reactions are essentially frozen, while the surface reactions are very fast. Nevertheless, in the boundary-layer flow over a highly cooled flat plate, gas phase recombination reactions are expected to occur even close to the leading edge.

For this reason numerical calculations including variable transport properties and nonequilibrium gas phase reactions were carried out for the flow of dissociated pure nitrogen over a

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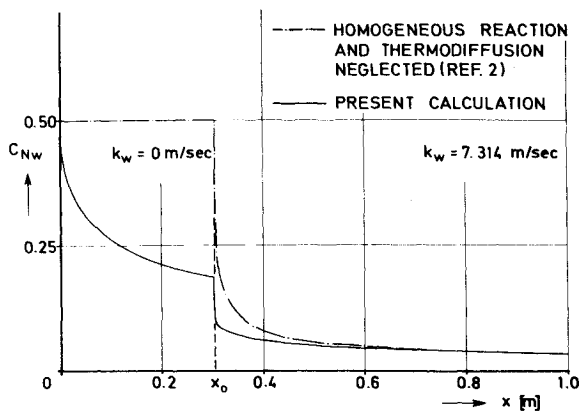


Fig. 1 Wall atom concentrations on a flat plate with discontinuously changing catalcity.

flat plate using the same boundary conditions as Hayday and McGraw. Details of the calculations may be found elsewhere.<sup>3</sup>

Figure 1 shows the comparison of the results of Hayday and McGraw, where gas phase recombination reactions and thermodiffusions were neglected, and the present calculations for the same flow parameters (freestream velocity  $u_\delta = 2438$  m/sec, freestream temperature  $T_\delta = 6000$  K, freestream nitrogen atom concentration  $c_{N\delta} = 0.5$ , and pressure  $p_\delta = 0.1$  atm). The wall atom concentration is plotted over the distance from the leading edge. As thermodiffusion was included in the calculations, the wall atom concentration decreases because of the high temperature gradient close to the leading edge to about  $c_{Nw} = 0.47$ . In the first part of the plate, which was assumed to be noncatalytic, the wall atom concentration continues to decrease due to gas phase recombination reactions to about  $c_{Nw} = 0.18$  at  $x = x_0 = 1$  ft. At this location the catalcity of the plate was assumed to change discontinuously to a finite value. A first-order heterogeneous reaction was assumed at the surface such that the boundary condition of the atom concentration equation is given by

$$\rho D_N (\partial c_N / \partial y) + (D_N^T / T) (\partial T / \partial y) = \rho k_w c_N \quad (1)$$

In this equation  $\rho$  denotes the density,  $T$  the temperature,  $D_N$  and  $D_N^T$  the binary diffusion and thermodiffusion coefficients, respectively,  $y$  the normal distance from the plate, and  $k_w$  the catalytic efficiency. A value of  $k_w = 7.314$  m/sec was chosen according to Ref. 2. In the second part of the plate the recombination is rapidly accelerated by the heterogeneous reaction. The wall atom concentration deviates strongly from the frozen flow values in both parts of the plate up to  $x \sim 2x_0$ . It is evident that gas phase reactions should not have been neglected in the calculations of Hayday and McGraw.

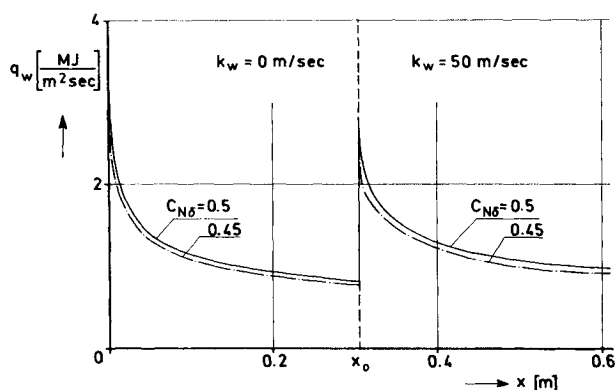


Fig. 2 Heat-transfer rate to the wall with the noncatalytic surface placed in front of the highly catalytic surface.

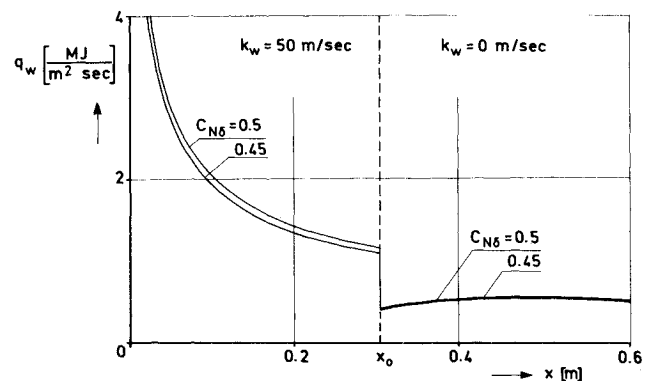


Fig. 3 Heat-transfer rate to the wall with the highly catalytic surface placed in front of the noncatalytic surface.

For a practical application, one would measure the integral heat transfer  $\bar{q}$  over a finite distance in each part of the plate

$$\bar{q} = \int_{x_1}^{x_2} q_w(x) dx \quad (2)$$

From the difference of heat transfer in the catalytic and in the noncatalytic part, the freestream atom concentration could be determined, if the function

$$c_{N\delta} = f(\Delta \bar{q}) \quad (3)$$

is known for a given arrangement.

In order to study the sensitivity of the double-surface gage to a change in freestream atom concentration, two different possibilities were investigated. Calculations of the heat-transfer rate  $q_w$  to the wall based on the same flow parameters as above but with a catalytic efficiency of  $k_w = 50$  m/sec are shown in Figs. 2 and 3. The freestream concentrations of atomic nitrogen were  $c_{N\delta} = 0.5$  and  $c_{N\delta} = 0.45$ , respectively. If the noncatalytic surface is placed in front of the highly catalytic surface (Fig. 2), the heat transfer to the wall is nearly the same in both parts of the plate and its difference is thus very insensitive to the change in freestream atom concentration. When the arrangement is reversed, (Fig. 3), a difference in heat transfer appears in the highly catalytic first part only. In this case a calculation of the integral heat transfer between  $x = 0.15$  m and  $0.25$  m as well as between  $x = 0.35$  m and  $0.45$  m for both concentrations of  $c_{N\delta} = 0.45$  and  $c_{N\delta} = 0.5$  shows that a change of 10% in the concentration corresponds to a change of 5% in the difference of the integral heat transfer  $\Delta \bar{q}$ . Thus the slope of the function Eq. (3) is very steep and its sensitivity would probably not be satisfactory for practical purposes.

Hence it is concluded that as a result of the influence of homogeneous reactions, the proposed double surface gage would generally not be suitable for a precise determination of freestream atom concentrations by heat transfer measurements in hypersonic laboratory facilities.

It should be mentioned that restrictions to the validity of the boundary-layer assumption at the leading edge of the plate and at the location of discontinuously changing catalcity were not taken into account in the calculations.

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